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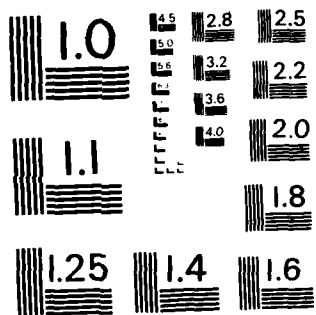
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FINAL SCIENTIFIC REPORT

GRANT NO: AROSR 81-0052, 81-0052A, 81-0052B

PERIOD: 1 January 1981 to 30 April 1984

PRINCIPAL INVESTIGATOR:

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PROJECT DESCRIPTION

Grant No. AFOSR 81-0052 is to study the numerical and analytic properties and solution of implicit systems of differential equations in the form,

$$A(t)x'(t) + B(x(t)) = f(t) \quad (1)$$

and their application to circuit and control problems. Here x is a vector state variable, A is a singular matrix and B is a (possibly) nonlinear vector valued function of the vector variable x .

PUBLICATIONS

During the report period the following papers were written as part of this research effort;

- P1. S. L. Campbell, Numerical procedures for analyzing nonlinear semistate equations, Proceedings Fifth International Symposium on the Mathematical Theory of Networks and Systems, 1981, 22-27.
- P2. S. L. Campbell, Consistent initial conditions for singular nonlinear systems, Circuits Systems and Signal Processing 2(1983), 45-55.
- P3. S. L. Campbell, Linear time varying singular systems of differential equations, 1981 (appears as a report and as Chapter 5 of [C3]).
- P4. S. L. Campbell, Nonlinear singular systems of differential equations, 1981 (appears as a report and as Chapter 6 of [C3]).
- P5. S. L. Campbell, The Drazin inverse of an operator, Recent Applications of Generalized Inverses, Pitman Publishing Co., 1982, edited by S. L. Campbell, 250-260.

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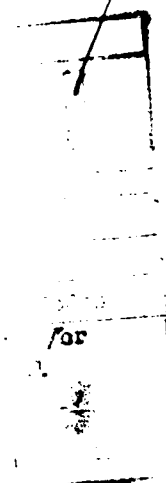
- P6. S. L. Campbell and K. Clark, Cheap control and nonsymmetric Riccati equations, 1981 (appears as a report and as Section 6 in Chapter 1 of [C3]).
- P7. S. L. Campbell and K. Clark, Order and the index of singular time invariant linear systems, Systems and Control Letters 1(1981), 119-122.
- P8. S. L. Campbell and N. J. Rose, A second order singular linear system arising in electric power systems analysis, Int. Journal Systems Science 13(1981), 101-108.
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- P10. S. L. Campbell, On positive controllers and linear-quadratic optimal control problems, International J. Control 36(1982), 885-888.
- P11. S. L. Campbell, One canonical form for higher index linear time varying singular systems, Circuits Systems & Signal Processing, 2(1983), 311-326.
- P12. S. L. Campbell and L. Petzold, Canonical forms and solvable singular systems of differential equations, SIAM J. Alg. & Discrete Methods 4 (1983), 517-521.
- P13. S. L. Campbell, The numerical solution of higher index linear time varying singular systems of differential equations, SIAM J. Sci. and Stat. Computing (to appear).
- P14. S. L. Campbell, Regularizations of linear time varying singular systems, Automatica (to appear).
- P15. S. L. Campbell, Multiparameter singular systems: Nondissective approaches, Proceedings 22nd IEEE-SIAM Conference on Decision and Control, Dec., 1983, 1278-1281.
- P16. S. L. Campbell, The numerical solution of differential algebraic equation systems, All About Simulators, 1984. Edited by V. Amico and A. Clymer, Society for Computer Simulation, Vol. 14, No. 1, 7-12.
- P17. S. L. Campbell and J. Rodriguez, Bounded solutions of discrete singular systems on infinite time intervals, IEEE Trans. Automatic Control, (to appear).
- P18. S. L. Campbell and J. Rodriguez, Nonlinear singular systems and contraction mappings, Proceedings 1984 American Control Conference (to appear).
- P19. S. L. Campbell, Non-BDF methods for the solution of linear time varying implicit differential equations, Proceedings 1984 American Control Conference (to appear).

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MATTHEW J. KEMPER
Chief, Technical Information Division

In what follows [Pi] denotes one of the above papers, [Ci] denotes another work of the proposer, and [Ri] denotes the work of other researchers. The [Ci] and [Ri] papers are listed at the end of this report.



RESULTS

Subsequent to writing the proposal for this contract, but prior to 1 January 81, the proposer showed [C4] that for a connected circuit made up of linear capacitors, (non-linear) current controlled resistors, and independent voltage sources, that the singular behavior of (1) was contained in a singular subsystem of the form

$$A\dot{x} + Bx = f \quad (2)$$

where A, B were constant matrices. This subsystem was not decoupled from the remaining nonsingular system. Nonetheless it was possible to use the theory for (2) to analytic advantage. Subsequently in [P1], the numerical implementation of the procedure was discussed.

Let $\nabla_x B$ at $x(t_0)$ be the Jacobian (vector gradient) of $B(x)$. Then the index of (1) at $x(t_0)$ [P2] is the index of nilpotency of $(\lambda A + \nabla_x B)^{-1} \nabla_x B$. The assumption that such a parameter λ exists is not only quite mild but necessary if numerical methods, such as backwards differences, are to be used on (1). For (2), $\lambda A + B$ being invertible is equivalent to solutions, when they exist, being uniquely determined by consistent initial conditions [C1], [C2]. In any case the index is independent of λ [C2]. If the system (1) is rewritten as an implicit or constrained system,

$$\dot{x} = H(x, y) \quad (3a)$$

$$0 = G(x, y) \quad (3b)$$

then the index being greater than one is equivalent to $\nabla_y G$ being singular [P2]. The vast bulk of the theory of nonlinear singular systems is devoted to the index one case. In particular most numerical methods assume the system is index one [R8]. Yet many of the circuits not admitting state

equations have operating points where the index is greater than one and it is at these places that jump behavior may occur. Even for (2), the index may be greater than one if operational amplifiers are present [R19]. One major thrust of this research effort is to understand these higher index cases.

In [P2], the consistent initial conditions to (1), that is those not admitting impulsive behavior, were characterized. It was also shown that for index ≥ 3 systems that this characterization was different from and corrected previously published results by other researchers in the circuit theory literature [R14]. An example was given to show these other characterizations were, in fact, incorrect. An important consequence of this paper is that if the index is greater than or equal to three at a point then a numerical method based on linearization such as an Eulers, may not only give a poor approximation but may actually jump from one solution to another.

Singular systems often arise as reduced order models in singular perturbation problems. In fact, one way to justify the distributional or impulsive solutions to (1) is through singular perturbation arguments [C3], [R3, R7]. The results of [P5] may be used in determining asymptotic expansions [C3] for linear systems with (2) as the reduced order model.

In [P7] it is shown that if the quadratic control problem has process

$$\dot{x} = Ax + Bu, \quad x(0) = x_0, \quad t_f \text{ specified}, \quad (4a)$$

and cost

$$J[x,u] = \frac{1}{2} \int_{t_0}^{t_f} x^T P x + u^T R u \, dt \quad (4b)$$

with R singular (i.e. a cheap control problem) then the index k of the Euler-Lagrange equations for (4) which take the form (2), is actually $2r + 1$ where the control problem (4) is what is classically called singular of order r

and of distributional order $k-1$. The implications of [P2], [P3], [P4] for higher order singular arcs in nonlinear cheap control problems remains to be determined but appears interesting.

If R in (4b) is invertible, then matrix Riccati equations are often used to solve and analyze the optimal control problem. If R is singular, then not all initial conditions admit continuous or nonimpulsive solutions. It is of some interest to determine the continuous solutions. In [P6] nonsymmetric Riccati equations are derived for the free end point problem that determine all continuous solutions. Numerical results are given which suggest that these nonstandard Riccati systems may be numerically more reliable than the more obvious Riccati equations. Why this occurs is discussed.

Of course, a second order singular system

$$M\ddot{x} + C\dot{x} + Kx = f \quad (5)$$

may be written as a first order system (2). However it is of some interest to derive solutions in terms of M , C , K . In general this is quite difficult and the general problem is still not satisfactorily solved. In large-scale interconnected electric power systems, the damping matrix C is usually assumed to have the form of Raleigh damping, that is, $C = \alpha M + \beta K$. Also because of restoring forces and the nonconservative nature of circulatory forces due to transfer inductances, M and K are not only not necessarily symmetric but are fairly arbitrary [R1]. In [P8], closed forms for the solutions of (5) with M singular and C in the form of Raleigh damping are derived.

Singular systems arise frequently in applications. However, researchers frequently are unaware of the work of others and as a consequence often

must rederive a portion of what is already known. For example, work on singular systems has been done under the names: descriptor, implicit, algebraic, constrained, semi-state, singular, reduced order, and degenerate; and in the fields of mathematics, systems theory, control theory, circuit theory, economics, power systems, numerical analysis, and biology. The need for a research level monograph on nonlinear singular systems that would tie together this work, have a comprehensive bibliography, and present new results became apparent to the proposer. Accordingly [C3] was written during 1981. Most of the results of [P7], [P8] are not discussed in [C3].

Chapters 5 and 6 of [C3] contain some results, ideas, and applications of this research effort that will not be published elsewhere. Accordingly, these chapters (slightly modified) were made available in the form of the reports [P3], [P4] to facilitate their distribution and utilization by the U.S. Government research and development agencies.

Numerical experiments had shown that in those cases when a first order method failed on (1) or the linear time varying problem

$$A(t)x'(t) + B(t)x(t) = f(t), \quad (6)$$

that a second order method sometimes worked but not with second order accuracy near points of higher index. Two results in [P3], [P4] shed light on this. First, in [P4] it was shown that whether the system was nonsingular, index one, or higher index was an invariant of the system under time varying coordinate changes. Secondly, in [P3] it was shown that for (6) a second order fixed step backwards difference scheme applied to essentially index one systems was able to integrate through points of higher index provided there were solutions through that point. Proofs were given and numerical

experiments agreed with the predicted error estimates. It is current numerical practice to use variable step methods on stiff equations which are index one. However, near index changes there are advantages to fixed step methods [C3], [R16], [R20] so that our analysis has concentrated on determining the behavior of fixed step methods.

There seem to be four basic types of questions to consider for any form of singular system.

Analytic Properties. Of course, there are the standard types of questions present whenever differential equations are studied. When and in what sense do solutions exist? For what initial conditions are there solutions? How can these solutions be explicitly characterized? Can the solutions be written in terms of solutions of explicitly given nonsingular problems?

Numerical Solutions. Given that solutions exist, how can one compute them directly from (1) as opposed to trying to solve (1) analytically and then numerically computing that answer.

Regularization of (1) is the problem of determining an equation

$$F(\epsilon, \dot{x}_\epsilon, x_\epsilon, t) = 0 \quad (7)$$

so that $F(0, \dot{x}_0, x_0, t) = 0$ is (1) and $x_\epsilon \rightarrow x_0$, in some sense, as $\epsilon \rightarrow 0^+$.

System (7) is called a regularization or regularizing perturbation of (1).

If $\nabla_y G(x, y)$ is stable (eigenvalues have negative real part), the usual regularization of (3) is the traditional singular perturbation problem

$$\begin{aligned} \dot{x}_\epsilon &= H(x_\epsilon, y_\epsilon) \\ \epsilon \dot{y}_\epsilon &= G(x_\epsilon, y_\epsilon). \end{aligned} \quad (8)$$

Form Variations. Systems will usually not be exactly in a specific form. Therefore it is important to know what types of transformations do not alter the basic properties of the form, and what systems are related by these types of transformations to the form.

In [P14], it is also shown that if (6) is in a special type of time varying form, denoted SCF, then the pencil perturbation is a regularization.

The SCF form is investigated in considerably more detail in [P11]. In [P11] a safe transformation on (6) is defined to be left multiplication by an invertible time varying matrix function $P(t)$ and a coordinate change $x = Q$ where Q is a constant invertible matrix. Not all systems (6) can be safely transformed into SCF [P11]. In [P11], these systems that can be safely transformed are characterized. They have several properties. First, the pencil perturbation is a regularization. Second, the analytic solution is given explicitly. In general, an implicit backwards difference scheme may not work on (6) [C2]. However, in [P11] it is shown that a fixed step implicit backward Euler's method will work on systems which can be safely transformed to SCF. This result may be extended to higher order methods by extrapolation. In [R6], Newcomb gives an example of a circuit involving a gyrator which is difficult to analyze by conventional means due to the presence of several parasitics which still leave one with a singular system in the form of (6). In [P15], this circuit is analyzed by the methods developed in [P11].

It is also of interest to develop analytic techniques which show how the solutions of (6) depend on A, B . In [P9], the index two case of (6) is solved analytically under a rank condition. This is the first solution of this type to appear in the literature. The results of [P9] are applicable

to a large number of systems that cannot be put into SCF and for which the pencil regularization does not work. It is worth noting, that neither [P9] nor [P11] assume that A in (6) has constant rank or constant index.

One problem in which higher order singular systems often occur is the reduced order model in a cheap control problem [C1], [C2], [P7]. In [R15], the linear control system:

$$\dot{x} = Ax + Bu, \quad x(0) = x_0, \quad 0 \leq t \leq t_1, \quad t_1 > 0 \quad (11)$$

$x, x_0 \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ with quadratic cost

$$J[u, x_0] = \frac{1}{2} x^T(t_1) H x(t_1) + \frac{1}{2} \int_0^{t_1} u^T R u + x^T Q x \, dt \quad (12)$$

was studied. Here H, Q, R are real symmetric matrices. The controls were real, bounded, and measurable with values in the positive orthant, \mathbb{R}_+^m . The two essential assumptions were, R is "strictly co-positive";

$$\text{There exists } \gamma > 0 \text{ so that } u^T R u \geq \gamma u^T u \text{ all } u \in \mathbb{R}_+^m, \quad (13)$$

and "Assumption A";

$$\min_{u \in \mathbb{R}_+^m} \left(\frac{1}{2} u^T R u - \gamma^T B u \right) \text{ has a unique solution for all } \gamma \in \mathbb{R}^n. \quad (14)$$

Conditions (13) and (14) do not appear to force R to be invertible. This suggests that the approach of [R15] might be useful in studying "cheap control" problems, that is, those for which R is singular. This possibility was investigated in [P10]. Unfortunately, it was shown that for almost all cases of interest conditions (13) and (14) force R to be positive definite and hence invertible.

The verification that implicit, fixed step backwards difference schemes work on constant coefficient systems [P20] and linear time varying systems in SCF [P11] depends on the existence of a canonical form. Even if the transformation to SCF alters the numerical properties of the system, the properties of the SCF developed in [P11] combined with information on the transformation may suggest modifications on the difference schemes that will enable them to be applied to the original system (6).

With these ideas in mind Gear and Petzold have been examining canonical forms for singular linear time varying systems [R9], [R10], [R11], [R16]. The form SCF developed in [P11], [P14] is considerably more general than the one suggested in [R9], [R10] and includes more systems of interest while maintaining the properties of interest. Also [R9], [R10] do not anticipate the value of considering safe transformations. There is also an error in [R10] which is corrected in [P12].

On a more theoretical level, there had not been much success in proving the existence or the uniqueness of solutions to (1) in higher index cases by functional analytic methods (fixed point theorems and contraction mappings), although some work had been done on the essentially index one case [R4], [R5]. The difficulty is that the standard Banach spaces and norms rarely work.

The principal investigator and a colleague, J. Rodriguez examined these questions. For the continuous case they developed a family of Banach spaces on which it was possible to construct contractions [P18]. The special structure required by these mappings helps explain why the traditional approach has met with limited success. For discrete nonlinear singular systems the situation is quite different [P17]. It is shown in [P17] that

not only do solutions exist on infinite time intervals but they may be approximated, up to a discrete boundary layer, by the solution of a finite time problem.

With the discovery that backward differences need not work on higher index problems, two important questions have been addressed. One has been to show that for certain classes of applications that the backward differences will converge. Positive results in this direction have recently been derived in [R2], [R11], [R12], [R13]. The other question is the search for a numerical method that will work even when the backward differences do not. The first such method for the linear time varying problem was developed in [P13]. This method, which is a type of Taylor series method, works in all cases for which the linear time varying system is known to be solvable. The analysis of this method and the best way to implement it are still under investigation. At each step it is necessary to solve a singular linear system in the least squares sense. Recently it has been found that some of the coefficients of this system can be estimated to lower accuracy [P19].

The considerations which effect the choice of numerical method for a solution of an implicit differential equation in a particular application are discussed in [P16].

Summary of Research Accomplishments

While there were several results derived during this contract, the major achievements may be summarized as follows.

1. The introduction and beginning analysis of the first general numerical method for the solution of (6), [P13], [P16], [P19].
2. A better understanding of what causes backward differences to fail to converge [P1], [P2], [P11].
3. A better structure theory for singular linear time varying and non linear systems [P3], [P4], [P9], [P11], [P12], [P14], [P15], [P17], [P18].
4. A beginning development of the interconnection between singular perturbations and backward differences for higher index problems [P11], [P14].
5. The application of singular systems to control and circuit problems of interest to the engineering community [P4], [P6], [P7], [P8], [P10], [P15], [P16].
6. The development of explicit solutions for the analysis of particular problems and the testing of numerical methods [P9], [P11].

TRAVEL

The paper [P1] was presented at the Fifth International Symposium on the Mathematical Theory of Network and Systems, Santa Monica, California, 1981. At this meeting, the proposer first became acquainted with the work of Sastry, et. al. [R17], [R18]. This very interesting work showed that for index one non-linear systems that whether one used the type of solutions provided by the proposer's work or used a different type of solution depended on the relative importance of noise vs. parasitics. The usual theory of stochastic differential equations is not immediately available for singular systems of higher index but preliminary work is underway to try and determine if there is a viable theory for stochastic systems of index higher than one and if this theory can be used to analyze noisy descriptor systems.

Project funds were used to partially pay for a two and one-half week trip in July of 1982. The remaining funds came from North Carolina State University and the principal investigator. The trip had two primary purposes. One was to talk with and communicate results to electrical engineers and applied mathematicians interested in the types of problems being considered in this project. The second was to acquire additional insight into how these problems and the related problems of singular perturbations/reduced order modelling fit into current engineering theory and practice.

The trip encompassed two formal meetings and one informal one. The first formal meeting was the "Seminar on Singular Perturbations in Systems and Control" sponsored by UNESCO and IFAC and held at CISM (International Centre for Mechanical Sciences) in Udine, Italy. This meeting consisted of four, two hour talks a day and some discussion. The talks were all in depth and expository but at a fairly advanced level. The second formal meeting was the "IFAC Workshop on Singular Perturbations and Robustness of Control

Systems" which was held in Ohrid, Yugoslavia. The emphasis of this meeting was on recent research. An early version of [P14] was presented as an invited paper. There were a large number of participants interested in the types of questions discussed in [P14]. Both of these meetings were fairly intensive with many talks scheduled. A more relaxed and informal gathering was held at the Hotel Croatia in Cavtat, Yugoslavia, during the period between the two meetings. Many (25-30) participants of both meetings took part in this informal meeting.

The paper [P12] was presented at the 1983 Society for Industrial and Applied Mathematics (SIAM) National Meeting in Denver, Colorado, in June, 1983.

The paper [P15] was presented as a regular paper at the 1983 IEEE Conference on Decision and Control in San Antonio, Texas, in December. This meeting is jointly sponsored by SIAM.

Finally, [P16] was presented as an invited paper in the "Solution of Equation Systems" Session at the 1984 Simulators Miniconference in Norfolk, Virginia. This meeting gave an opportunity to discuss the results of this contract with a large group which is actively involved in the numerical modeling of flight, control, and process systems.

OTHER PROJECT PERSONNEL

Research Assistant:

Throughout the contract period, the research assistant was Mr. Kenneth Clark. Mr. Clark has a B.S. in Mathematics and an M.S. in Operations Research (both at NCSU) and is a Ph.D. candidate in Applied Mathematics at North Carolina State University. His thesis advisor is the principal investigator. Mr. Clark performed literature searches, programmed and ran most of the numerical studies, performed some of the more routine calculations, and provided an informed colleague to discuss the research with. During the last year of this contract he has been actively engaged in research on the relationship between regularization and convergence of implicit backward differences for implicit differential equations.

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